Users of coordinate measuring machine and large gantry machines need to ensure the volumetric performance of their machines in order to inspect, or machine, mechanical parts with precision. In the case of CMMs, the ASME B89.4.10360.2-2008 document imposes seven directions for a volumetric check. These directions offer some redundancy for axis location (out-of-squarenesses) and scale factor estimation. The paper looks at the opportunity of using the data for immediate machine correction and the risks involved. In particular, the fact of using test data for calibration and verification, the potential contamination from non-modelled motion errors and the representativity of the estimated parameters are considered. Experimental results obtained using laser interferometry on a LEGEX CMMs are used to present the main concepts.

1. INTRODUCTION

The coordinate measuring machines are often the preferred instrument for the dimensional and geometric control of parts. To ensure the correct operation of the CMM, it is necessary to check its performance periodically. The machine is affected by 21 geometric errors including 18 joint motion errors and three link errors axis to axis which can become significant over time. Experience suggests that the variations related to scale errors and out-of-squarenesses between axes are particularly at risk.

Verification and calibration of coordinate measuring machines has been the subject of several studies, different types of artifacts were used such as, gauge blocks, step gauges, ball plate, hole plate and ball bar [1-6]. G. Zhang et al. [7] modelled kinematic errors of CMM by 18 errors and added the 3 out-of-squarenesses between axes which are the coefficients of the linear term of straightness errors. Out-of-squarenesses are determined by measurements along face diagonals of the three planes XY, XZ and YZ. Kruth et al. [8] measure a non calibrated artefact in four body diagonals of the CMM and deduce the values
of the squareness errors. For verification of the performance of large CMM, say up to four metres of displacement, Phillips et al. [9] used an artefact equipped with a laser. A retro-reflector is secured to a sphere that is probed by the CMM, while the laser measures the displacement of the retro-reflector. The ASME B89.4.10360.2-2008 report [10] suggests the use of five lengths (which can be done with laser) measured in seven predefined positions to provide an indication of the performance of the CMM, but without assessing parametric sources from the machine. This study provides a method for determination of scale errors and out-of-squarenesses between axes using the data for a volumetric check.

2. ERROR OF POSITION

2.1. POSITION OF THE STYLUS TIP AND THE WORKPIECE IN COORDINATE SYSTEM

The machine used for this study is a MITUTOYO model LEGEX 9106 with topology WYFXZT. Fig. 1 illustrates the coordinate system (X, Y, Z) for the position of the centre $O_z$ of the articulated system of the probe $t$ and the position of a point to measure on the workpiece $w$. These coordinates are defined in reference frame $\{F\}$ and calculated from the movement of the carriages of the machine. The coordinates of the stylus tip $t$ ($x_t$, $y_t$, $z_t$), in the reference frame $\{F\}$, are defined by the moving axes coordinates X and Z, by the length $L_s$ (distance from the pivot of the articulated system to the centre of the stylus tip) and the orientations of the articulated system (angles $A$ and $B$). The coordinates of a point to measure on the workpiece $w$ ($x_w$, $y_w$, $z_w$), in the reference frame $\{F\}$, are defined by the moving axes coordinates Y.

![Fig. 1. Position of the stylus tip and workpiece in the coordinate system](image)

On a real CMM, the positions of the stylus tip and workpiece do not exactly correspond to the nominal position; this position is affected by the deviation related to the joints of the machine. Fig. 1 shows the linear and angular deviation associated to each joint.
X, Y and Z; For example. $\tilde{\delta}_X$ and $\tilde{\varepsilon}_X$ are respectively the linear and the angular deviation associated to the X joint.

2.2. ERROR POSITION OF THE STYLUS TIP AND THE WORKPIECE

The position of the stylus tip and the workpiece is affected by the deviation associated to the joints. Angular deviation spread in the body rigid structure by the effect of the Abbé offsets. The linear deviation has a direct effect on the position of stylus tip and the workpiece. The position error is the deviation between the real and the nominal position.

The position error of the stylus tip is [11]

$$\vec{e}_t = \tilde{\delta}_x + \varepsilon_x \wedge \vec{O}_x t + \tilde{\delta}_z + \varepsilon_z \wedge \vec{O}_z t$$

(1)

Similarly, the position error of the workpiece is

$$\vec{e}_w = \tilde{\delta}_y + \varepsilon_y \wedge \vec{O}_y t$$

(2)

The volumetric error that characterizes the position error of stylus tip relative to the workpiece is:

$$\vec{e}_M = \vec{e}_t - \vec{e}_w$$

(3)

Assuming the scale errors and out-of-squarenesses are dominant. After decomposition of vectors $\vec{O}_x t$, $\vec{O}_y t$ and $\vec{O}_z t$ in the frame \{F\} we have:

$$\vec{e}_M = \begin{pmatrix} K_x \cdot X \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \varepsilon_{x,X} \\ \varepsilon_{y,X} \\ 0 \end{pmatrix} \wedge \begin{pmatrix} t_x \\ t_y \\ Z + t_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ K_z \cdot Z \end{pmatrix} - \begin{pmatrix} \varepsilon_{z,X} \\ \varepsilon_{z,Y} \\ 0 \end{pmatrix}$$

(4)

$$x_i = X + t_x$$
$$y_i = t_y$$
$$z_i = Z + t$$

(5)

$K_x$, $K_y$ and $K_z$ are the scale gain errors of the X, Y and Z axis respectively. $\varepsilon_{z,Y}$, $\varepsilon_{z,X}$ and $\varepsilon_{x,X}$ are the out-of-squarenesses of the Y-axis relative to X-axis, the Z-axis relative to X-axis and
the Z-axis relative to Y-axis respectively.

The nominal coordinates of the stylus tip relative to frame \( \{ F \} \) are:

In matrix form, equation (4) is

\[
\bar{e}_M = J \cdot \delta P
\]  

\[
\begin{bmatrix}
- \delta Y & 0 & 0 & 0 \\
0 & - \delta Z & 0 & - \delta Y \\
0 & t_x & - \delta t_y & 0 \\
0 & 0 & - \delta t_z & 0
\end{bmatrix}
\begin{bmatrix}
\epsilon_{x,y} \\
\epsilon_{x,x} \\
K_x \\
K_y \\
K_z
\end{bmatrix}
\]

3. GENERAL EXPRESSION OF ERROR OF MEASUREMENT OF DISTANCE

A linear laser interferometer is used to acquire the performance data. The measurement line \( i \) in the coordinate system of the CMM is defined by the two angles, \( \alpha_i \) and \( \beta_i \), and the coordinates of the first point \( M_{1,i} \), shown in Fig. 2. The coordinates of the nominal points \( M_{j,i} \) in the line of measurement are:

\[
\overline{O_jM_{j,i}} = O_jM_{1,i} + M_{1,i}M_{j,i}
\]

where:

\[
\overline{M_{1,i}M_{j,i}} = \|M_{1,i}M_{j,i}\| \cdot \hat{n}_i \quad \text{with} \quad \hat{n}_i = \begin{pmatrix}
\cos(\beta_i) \cos(\alpha_i) \\
\cos(\beta_i) \sin(\alpha_i) \\
\sin(\beta_i)
\end{pmatrix}
\]

The measurement error is the difference between the nominal target displacement and the real distance measured by the laser. From a modelling point of view, the measurement error \( E_{1,i,j} \) between the experimentally measured point \( M_{1,i} \) and the point \( M_{j,i} \) is the projection, in direction \( \hat{n}_i \), of the difference between the volumetric error calculated at point \( M_{j,i} \) and point \( M_{1,i} \).

\[
E_{1,i,j} = (\hat{e}_{u_j,i} - \hat{e}_{u_{1,i}}) \cdot \hat{n}_i
\]

Using the equation (6), equation (8) becomes

\[
E_{1,i,j} = \left( J_{j,i}^T \hat{n}_i \right)^T \delta P
\]
where $J_{j,i}$ is the jacobian matrix at a point $M_{j,i}$ for a measurement in the direction $\hat{n}_i$.

By measuring all target distance in the seven predefined positions by the AMSE report, a system (10) of equations is built, where each line corresponds to one measurement error:

$$E = H \delta p$$

(10)

where $E$ is a column matrix, comprising all the errors of measurements made and $H$ is the identification matrix.

4. EXPERIMENTS

4.1. MACHINE AND LASER SETUP

The experimental measurements set up on a MITUTOYO LEGEX 9106 installed in the dimensional metrology laboratory of Ecole Polytechnique in Montreal is shown in Fig. 3. Fig. 4. shows the seven positions predefined by the ASME report for periodic verification. A Renishaw ML10 Gold Laser was used to measure the actual displacement.
Fig. 3. Photo of the setup

Fig. 4. Seven measurement positions predefined by the ASME report

4.2. RESULTS FOR THE SEVEN PREDEFINED POSITIONS

Fig. 5 shows the results of measurements. Each curve represents the mean of 3 repetitions. The curves are the model prediction using the following estimated scale gain error and out-of-squarenesses:

- out-of-squarenesses: $\varepsilon_{x,y} = 2.7 \, \mu\text{rad}$, $\varepsilon_{y,x} = 4.2 \, \mu\text{rad}$ and $\varepsilon_{x,x} = 2.5 \, \mu\text{rad}$

- scale gain errors: $K_x = 1.7 \, \mu\text{m/m}$, $K_y = -4.8 \, \mu\text{m/m}$ and $K_z = -0.74 \, \mu\text{m/m}$.

A percentage $D$ representing the maximum residual $R_{max}$, relative to the maximum error $E_{max}$, is calculated, this percentage characterize the ability of the identified parameters to correct the machine.
Fig. 5. Results of measurements in seven positions predefined by ASME report

\[ D = \left(1 - \frac{R_{\text{max}}}{E_{\text{max}}} \right) \times 100\% \]  

(11)

For the measurement in the seven predefined positions presented in Fig. 5, \( E_{\text{max}} = 7.4 \, \mu m \) and \( R_{\text{max}} = 0.75 \, \mu m \), so \( D = 89\% \). That means 89% of the maximum error will be corrected using the six identified parameters and the maximum error after correction is estimated at 0.75 \( \mu m \).

To consider the representativity of the estimated parameters, among the seven measurement positions, only six positions are used to identify the six parameters and the results are used to predict the measurement in the positions not used for the identification of parameters. Fig. 6 presents the measured and predicted errors for the position not used for identification, for the seven studied cases.

Two examples are presented:

Example 1: positions 1, 2, 4, 5, 6 and 7 are used for the identification; 3 is excluded. Fig. 6 presents the measured and predicted errors in position 3. For this position, a maximum residual \( R_{\text{max}} = 1.2 \, \mu m \) and a maximum error \( E_{\text{max}} = 7.4 \, \mu m \), for \( D = 83\% \).

Example 2: positions 1, 2, 3, 4, 5 and 6 are used for the identification; 7 is excluded. Fig. 6 presents the measured and predicted errors in position 7. The maximum residual, in this position, is \( R_{\text{max}} = 0.92 \, \mu m \) and the maximum error is \( E_{\text{max}} = -0.28 \, \mu m \), for \( D = -228\% \). That means that the error in position 7 becomes larger than before correction. The maximum error, in position 7, after correction, is estimated at 0.92 \( \mu m \).
Fig. 6. Results of measurements in seven predefined positions and prediction using estimated parameters with six positions

Fig. 7. Measurement positions a) 7 measurement positions predefined by ASME report; 24 measurement positions in a) 3D view; b) 2D view (special proposed representation)
Because the directions used to check the CMM are also used for calibration, the CMM is optimised for the performance check. This should not be considered good practice even though it saves time to the user. In order to verify the ability of the six parameters identified using the data from a volumetric check with seven predefined positions, 24 positions (Fig. 7a and Fig. 7b) are used to predict the measurements in the volume of the machine. The Renishaw ML10 Gold Laser was mounted on a rotating module and placed in the middle of the machine table. This allowed to direct the laser beam successively towards the four corners of the table and, by using a flat mirror, to redirect it in 24 measured positions without dismounting the laser, which shortened measurement performing time significantly by greatly simplifying the setup changes.

4.3. VALIDATION USING ANOTHER 24 POSITIONS

Fig. 8 shows a comparison between the errors measured and the errors predicted by calculation in the 24 measured positions others than the 7 positions predefined by the ASME report and used for the model estimation.
Fig. 8. Result of measurement in 24 positions other than 7 used for identification of six parameters; a) from 8 to 13 b) from 14 to 19 c) from 20 to 25 d) from 26 to 31 measurement positions

It shows that the maximum measured error is 5.5 µm and the maximum residual is 2.6 µm, so $D = 52\%$. The percentage of error non-explained by the model may be due to the measurement uncertainty and the potential contamination from non-modelled motion errors [12].

It can be observed that the predicted errors depend only on the directions of measurement; for example: the prediction errors in the position 16 and 18 are the same. This is expected because only scale and out-of-squarenesses are modelled.

If scale gain errors are dominants the measurement errors in two positions parallel to the tested axes (for example position 20 and 21) should be identical. However, as can be seen in Fig. 8c) these measurement errors are different. This suggests that other errors sources are present and are significant. This shows the limits of the model predictive capability.
5. CONCLUSIONS

The method proposed in this paper identifies scale gain errors and out-of-squarenesses between axes of a CMM. Displacement measurements are taken with a linear laser interferometer in the seven positions specified in ASME B89.4.10360.2-2008 report.

The representativity of the estimated parameters from data for a volumetric check using six positions, at time, among all seven is evaluated by predicting the measurement error in directions not used for the identification process.

The laser head is swivel mounted centrally on the CMM table which allows creating 24 directions from the 4 bottom corners of the measuring volume.

The advantage of this approach is that it is fast and simple to implement; it provides parameters necessary to make corrective actions to the machine after doing a verification of the machine performance according to the ASME B89.4.10360.2-2008 report. This approach has also weaknesses, because there are potential contaminations from non-modelled motion errors. This approach is not limited to CMM, but it can also be applied to large gantry machine tools.

REFERENCES